CREST FACTOR, POWER FACTOR, AND WAVEFORM DISTORTION

An Environmental Potentials White Paper

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By definition, the crest factor of a voltage is equal to the peak value divided by the effective (rms) value. In the case of a sinusoidal voltage (which evidently has no distortion) the crest factor is $\sqrt{2} = 1.41$. A wave having a crest factor less than 1.4 tends to be flat-topped. On the other hand, a crest factor greater than 1.4 indicates a voltage that tends to be pointy.

By definition, the total harmonic distortion (THD) of current or voltage is equal to the effective value of all the harmonics divided by the effective value of the fundamental. In the case of distorted current, the equation is:

$$\text{Total harmonic distortion (THD)} = I_H/I_F$$

In the case of a distorted voltage, the THD is given by: E_H/E_F

From these expressions, it is seen that sinusoidal voltages and currents have a THD of zero.

The concept of power factor in the case of sinusoidal voltages and currents, relates to the real power, reactive power, and apparent power associated with a load consisting of resistance and reactance bringing about a direct phase shift between the voltage and current.

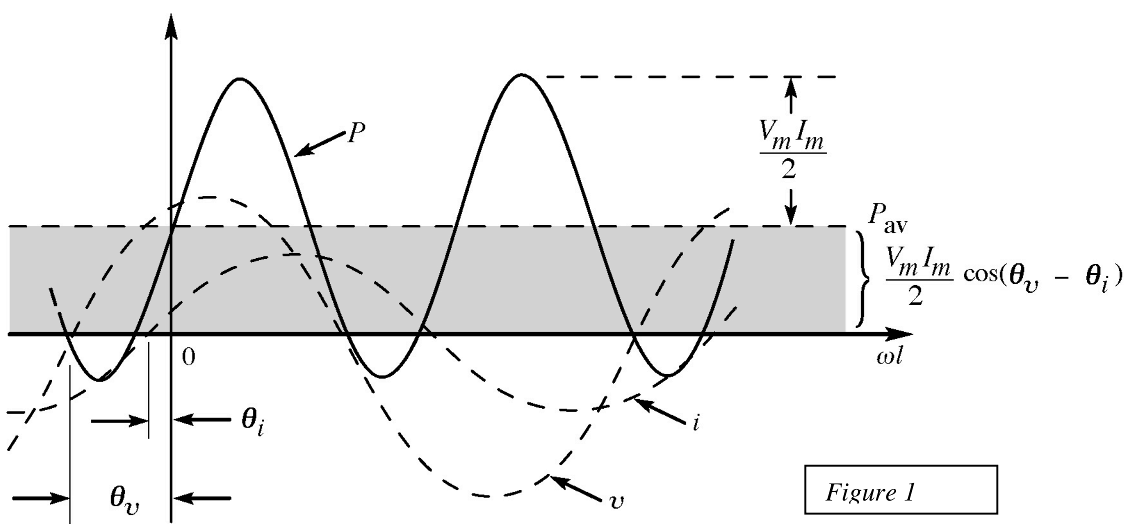

Figure 1

This concept is depicted in the above figure. For any load in a sinusoidal network, the voltage across the load and the current through the load will vary in a sinusoidal nature. In the general case,

$$V = V_m \sin(\omega t + \theta_v)$$

$$I = I_m \sin(\omega t + \theta_I)$$

Then the power is defined by

$$P = VI = V_m I_m \sin(\omega t + \theta_v)\sin(\omega t + \theta_I)$$

Using the trigonometric identity

$$\text{Sin A Sin B} = [\text{Cos}(A - B) - \text{Cos}(A + B)]/2$$

Now the product of I and V will result in a fixed value along with a time varying value of power so that

$$P = [(V_m I_m/2)\cos(\theta_v - \theta_I)] - [(V_m I_m/2)\cos(2\omega t + \theta_v + \theta_I)]$$

Fixed value Time varying (function of time)

A plot of V, I, and P on the same set of axes is shown in the figure above.

Note that the second factor in the preceding equation is a cosine wave with an amplitude of VmIm/2 and with a frequency <u>twice</u> that of the voltage or current. The average value of this term is zero over one cycle, producing no net transfer of energy in any one direction.

The first term in the preceding equation, however, has a constant magnitude (no time dependence) and therefore provides some net transfer of energy. This term is referred to as the **average power**, the reason for which is apparent from the figure above. The average power, or **real power** as it is sometimes called, is the power delivered to and dissipated by the load. It corresponds to the power calculations performed for dc networks. The angle $(\theta_V - \theta_I)$ is the phase angle between V and I, since $\cos(-\alpha) = \cos(\alpha)$.
The magnitude of average power delivered is independent of whether V leads I or I leads V. Defining θ as equal to the difference between θ_V and θ_I
We have $P = (VmIm/2)\cos\theta$ Using effective values for V and I this would be $P = V_{eff}\, I_{eff} \cos\theta$, in watts of real power delivered to and dissipated in a resistive load.

For a purely resistive circuit, since V and I are in phase, $|\theta_V - \theta_I| = 0° = \theta$, and cos 0° = 1, so that

$$P = (Vm\ Im)/2 = V_{eff} I_{eff}$$

Since $I_{eff} = V_{eff}/R$, then $P = (V_{eff})^2/R = (I_{eff})^2 R$.

However, in a purely inductive circuit, we find that the voltage drop across the terminals of the inductor (L) is given by $V = L(di/dt)$. It can be seen that induced voltage becomes a function of frequency, and V will lead I by 90°. Then

$$\theta_V - \theta_I = \theta = (-90°) = 90°$$

Therefore, $V = (V_{eff}I_{eff})\cos 90° = (V_{eff}I_{eff})\,(0) = 0$ watts

The average power or power dissipated by the ideal inductor (no associated resistance) is zero watts.

Now in a purely capacitive circuit, Ic is given by $I_c = C(dV/dt)$. In this case I leads V by 90°, so $\theta_V - \theta_I = \theta = (-90°) = 90°$ Therefore, as before

$$P = (V_{eff}I_{eff})\cos(90°) = 0 \text{ watts.}$$

The average power or power dissipated by the ideal capacitor (no associated resistance) is zero watts. In the equation $P = V_{eff}I_{eff}\cos\theta$, the factor that has significant control over the delivered power level is the $\cos\theta$. No matter how large the voltage or current, if $\cos\theta = 0$, the power is zero; if $\cos\theta = 1$, the power delivered is a maximum.

Since it has such control, the expression '$\cos\theta$' is given the name *POWER FACTOR*.

When the load is a combination of resistive and reactive elements, the power factor will vary between 0 and 1. If the current leads the voltage across the load, the load has a leading power factor. If the current lags the voltage across the load, the load has a lagging power factor.

In a circuit containing resistance and reactance, the product of the voltage and current is given the term 'APPARENT POWER' , and is represented symbolically by S

$$S = V\,I \quad \text{voltamperes (VA)}$$

Further, the combination of resistance, capacitive, and inductive reactance presents an IMPEDANCE $Z = R +/- jX$, where jX is the net reactance, depicted in the following figure:

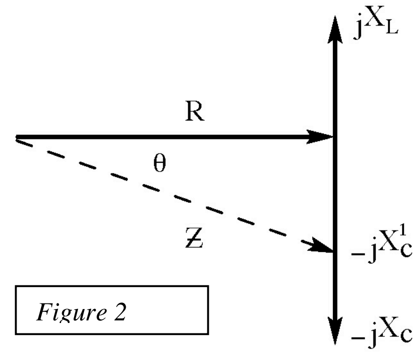

Figure 2

The impedance Z is given by $Z = R - jX_{NET}$ where $X_{NET} = -jXc'$

Z can be written as $Z = \sqrt{(R^2 - X'^2)}$. With Z, then, $V = I Z$ and $I = V/Z$.

$S = I^2 Z$ (VA) and $S = (V^2/Z)$ (VA)

Noted earlier, the average power delivered to the load is $P = V_{eff}I_{eff} \cos\theta$. However, $S = V I$, therefore, $P = S \cos\theta$. The power factor of a system is then given by $\cos\theta = P/S$, and is the ratio of the average (real) power to the apparent power.

It has been noted before that the net flow of power to the pure (ideal) inductor or capacitor is zero over a full cycle, and no energy is dissipated. The power absorbed or returned by the inductor or capacitor at any instant of time is called 'REACTIVE POWER', and is symbolized by 'Q'. Because of the 90° lagging or leading relationship,

$$Q = V I \sin\theta \quad \text{(Volt-ampere reactive, VAR)}$$

The three quantities AVERAGE POWER, APPARENT POWER, and REACTIVE POWER are related in a power triangle, depicted as follows:

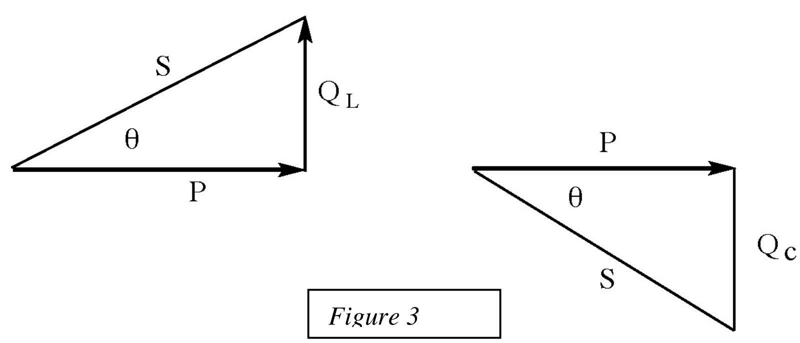

Figure 3

Since the reactive power and the average power are always angled 90° to each other, the three powers are related by the Pythagorean theorem

$$S^2 = P^2 + Q^2$$

Within a system containing distributed resistance and reactance, when impressed with VI voltamperes, the resistance will always absorb and dissipate $VI\cos\theta$ watts. However, the distributed reactance will store and <u>source</u> $VI\sin\theta$ vars. So, effectively, a reactive load will really become a 'generator', and the reactive power Q will reflect and circulate throughout the network loop until it is dissipated in the distributed resistance, or returned ultimately to the utility power grid.

Referring now back to the equation relating the fixed value of P : $P_f = V_{eff}I_{eff}\cos(\theta_V - \theta_I)$, and the time-varying value of P: $P_{tv} = V_{eff}I_{eff}\cos(2\omega t + \theta_V + \theta_I)$, we note that the complete expression for P is frequency sensitive. Voltages and currents in industry are often distorted. The distortion may be caused by magnetic saturation in the core of a transformer, by the switching action of thyristors, contactor switching, or any other non-linear load. A distorted wave is made up of a fundamental, related harmonics of the fundamental, and random high frequency noise produced by many coupled resonant waves within the network.

Therefore, in the above expression for the time dependent power will result in a rather complicated value for power, depending on $\cos(2\omega t)$. So, based on this fact, the meaning of 'power factor' must be enlarged upon when distorted voltages and currents are present. The terms *Displacement Power Factor* and *Total Power Factor* are then used.

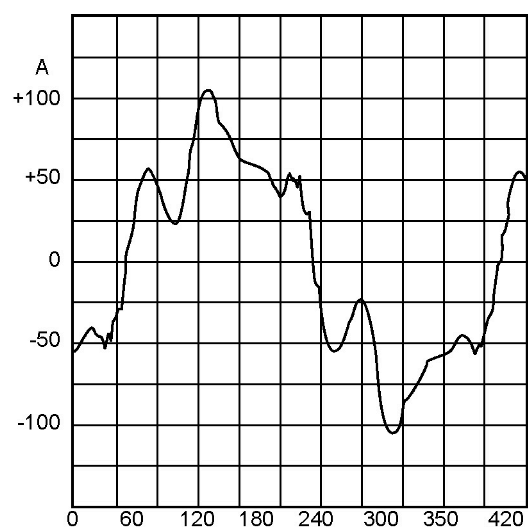

Figure 4

To illustrate *Displacement Power Factor*, note the figure above.

This is a waveshape of a distorted 60 Hz current having an effective value of 62.5 A. The current contains the following components: fundamental (60 Hz): 59 A; 5th harmonic: 15.6 A; 7th harmonic: 10.3 A. Higher harmonics are also present but their amplitudes are small, totaling 8.66 A.

The waveshape depends not only on the frequency and amplitude of the harmonics but also on their angular position with respect to the fundamental. For the above current wave, the effective (or rms) value of all the harmonics is calculated to be:

$I_H = \sqrt{(I^2 - I_F^2)}$
$= \sqrt{(62.5^2 - 59^2)} = 20.6$ A

The total distortion factor is then calculated to be

$$THD = I_H/I_F$$

$$= 20.6/59 = 0.349 = 34.9\%$$

The total effective value of the current in the circuit is:

$$I_T = \sqrt{\{(I_{60})^2 + (I_5)^2 + (I_7)^2 + (I_{HI})^2\}}$$

$$= \sqrt{(59^2 + 15.6^2 + 10.3^2 + 8.66^2)} = 62.5 \text{ A}$$

$$I_H = \sqrt{\{(I_T)^2 - (I_{60})^2\}} = 20.62 \text{ A}$$

Apparent power including high frequency components = E (62.5) VA

Reactive power including high frequency components = E (20.62) VARS

For this case, the phase angle will be arcs in (20.6/62.5) = 19.24°

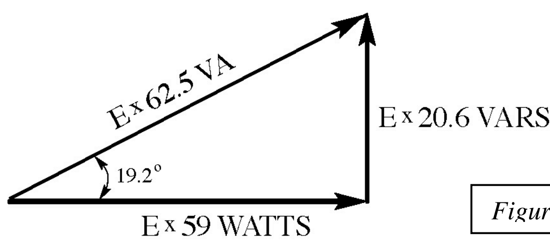

Figure 5

The average or real power will then be

$$P = E\ (62.5) \cos 19.24° = E\ (59) \text{ watts}$$

The power factor is 94.4 %, just based on high frequency effects alone.

On the other hand, considering a network with the above high frequency content, there will be a fundamental frequency (60 Hz) power factor related to and dependent upon the magnitude of vars, derived from <u>sinusoidal voltage and current</u> applied to linear reactive loads.

For example, let's say the power factor on that basis happens to be 98%. When we depart from sinusoidal waveforms and go to waveforms having harmonic and high frequency content, then the vars magnitude increases, as shown above, and the term *displacement power factor* is involved.

It is important to know how a circuit responds to harmonics and high frequency noise. In linear circuits composed of resistance, inductance, capacitance, transformers, and rectifiers, the various harmonics and high frequency noise components act independently of each other. Multiple resonances will develop within the network and the resultant frequency spectrum appears as wideband noise. But it can still be evaluated in a Fourier spectrum analysis, and will show a multitude of spectral lines, almost converging into a continuous curve.

As an example of an industrial power factor control problem involving high frequency noise, consider the following:

With the advent of digitally controlled systems, variable speed drives, and other capacitor input loads, the power line typically looks into a network containing rectifiers followed by capacitors. The rectifier essentially functions as a means to charge the capacitor each half cycle as shown below:

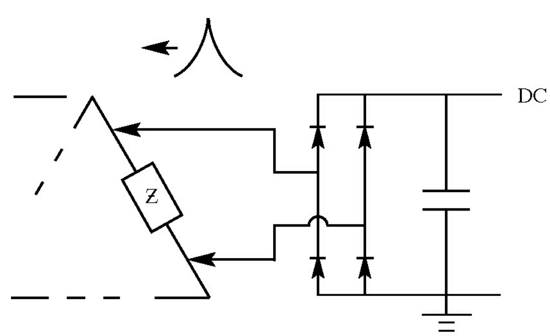

Figure 6
Reverse Current Spike

But when a voltage is applied across a capacitor, a large reverse current spike returns back through the rectifier, appearing as an addition to the line or branch current. A change in the branch or line current phasing or magnitude with respect to the line or branch voltage will have a ripple effect throughout the overall network.

The current spike travels back along the line and circulates in the loop. Since the spike constitutes a relatively high bandwidth (i.e., high frequency noise) wave, it will resonate with the distributed inductance and capacitance along the line and with any other fixed component within the network. This brings about high frequency ringing, and thus substantial electrical noise.

From the source, looking back into the line connecting the capacitor-rectifier load, the current as a repetitive spike each half cycle forces the overall phase with respect to the voltage to essentially increase the magnitude of the vars. This results in an effective ring back of volt amperes on the line. The source then sees this as a reduction of power factor.

From the following figure, it can be seen that the percentage of high frequency content of current compared to that of the fundamental is relatively high.

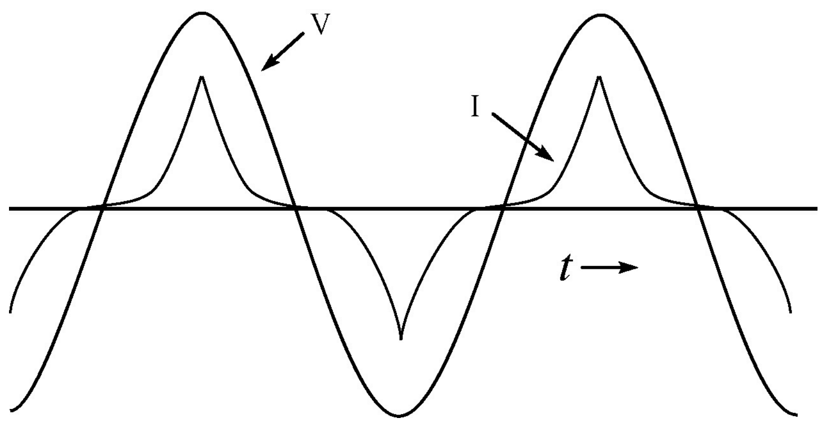

Figure 7

By attenuating the sharpness of the current spike the effective power factor can be improved. It is most important to filter and absorb these anomalies.